X Junior Balkan Olympiad in Informatics

Skopje, 2016

Day 2: Hacking tasks

Short description: Note that the given information defines a directed graph G(V, E), where V is the set of committee members. If there is an edge (u,v) in G, it means that u receives a message from v.

The solution is the following: If G(V,E) has a cycle of length >2 then print Benedictus. If G(V, E) has more than one cycle of length 2, print Armadillo. If G(V,E) has only one cycle of length 2, consisting of nodes u1 and u2, check whether all the edges apart from (u1, u2) and (u2, u1) are of the form (a, u1) or (a, u2). If it is true, then print Both, otherwise print Armadillo.

Analysis:

Note that because there is exactly one edge incident to each node, there is at least one cycle in the graph. Each of these 2 systems creates a specific graph, and our task is to find the form of the graph each system creates.

Lemma 1. The graph corresponding to the Armadillo system does not have a cycle of length greater then 2.

Proof: Assume that there is a graph corresponding to Armadillo system having a cycle $(a_1, a_2), (a_2, a_3), \ldots, (a_{n-1}, a_n), (a_n, a_1)$, of length $n > 2$. This implies that $\forall\ x \in V$, $d(a_i, a_{i+1}) \geq> d(a_i, x)$, $i=1,..n-1$, where $d(x, y)$ is a distance between the points x and y. Moreover, $d(a_i, a_{i+1}) >\geq d(a_i, a_{i-1}) = d(a_{i-1}, a_i)$.

Without loss of generality we can assume that $a_1 = \max_i\{a_i\}$. From the definition of the Armadillo system we have that:

$$d(a_n, a_1) > d(a_n, a_{n-1}) >\geq d(a_{n-1}, a_{n-2}) \geq> \ldots >\geq d(a_2, a_1) >\geq d(a_1, a_n) = d(a_n, a_1),$$

which is a contradiction.

Lemma 2: A graph corresponds to the Benedictus system if and only if it consists of a cycle C: $(a_1, a_2), (a_2, a_3), \ldots, (a_{n-1}, a_n), (a_n, a_1)$. All other edges have the form (x, a_i), where a_i is a node in C.

Proof: For the graph corresponding to Benedictus system we only care about the Y coordinate. Each edge is directed to the vertex with bigger Y, there is only one "back edge", from the leftmost point with the greatest Y coordinate, to the leftmost point with the smallest Y coordinate. The edge from the point (x, y) for which there is another point (z, y), z<x, has the same ending point as (z, y).

Let's assume now that we have a graph G(V, E) consisting of a cycle C and all the other edges are in the following form: (x, a_i), where a_i is a node in C. Then we may place the points into a coordinate (0, i), i=1,..,n. For each edge (x, a_i), $x \notin \{a_1, \ldots, a_n\}$, we will place the point (x, a_{i-1}), $i>1$ and (x, a_n), $i=1$.

Theorem. If G(V,E) has a cycle of length >2 then the graph corresponds to the Benedictus system. If G(V, E) has more than one cycles of length 2, then the graph corresponds to the Armadillo system. If G(V,E) has only one cycle of length 2, consisting of nodes u1 and u2, and all edges different from (u1, u2) and (u2, u1) are of the form (a, u1) or (a, u2), then then the graph can corresponds to both systems.

Proof: Since we are sure that the graph corresponds to either Armadillo or Benedictus, from Lemma 1 we have that whenever there is a cycle with length greater then 2, then it is corresponding to the Benedictus system. The last step to our solution is to prove that if G(V,E) has only one cycle of length 2, consisting of nodes u1 and u2, and all edges apart from (u1, u2) and (u2, u1) are of the form (a, u1) or (a, u2), then both systems can be constructed. It is clear from Lemma 2 that it is possible to construct Benedictus system from such a graph. We must also prove that it is possible to construct Armadillo as well. Let $V=\{a_1, \ldots, a_n\}$ and $E=\{(a_1, a_n), (a_n, a_1)\} \cup \{(a_i, a_n) | i \in A \subseteq \{2,\ldots,n-1\}\} \cup \{(a_i, a_1) | i \in \{2,\ldots,n-1\} \setminus A\}$. Then if we add the point (1,0) for the node a_1, the point (2n,0) for node a_n, for each a_i, $i \in A$ we add the point (2n-i, 0) and for each a_i, $i \in \{2,\ldots,n-1\} \setminus A$ we add the point (i, 0), Our graph follows the Armadillo system and therefore our proof is complete.